Quality and Statistical Process Control

Prof. Sid Sytsma – Ferris State University

The concept of quality has been with us since the beginning of time. As early as the creation of the world described in the Bible in Genesis, God pronounced his creation "good"-- e.g., acceptable quality. Artisans' and craftsmen's skills and the quality of their work are described throughout history. Typically the quality intrinsic to their products was described by some attribute of the products such as strength, beauty or finish. However, it was not until the advent of the mass production of products that the reproducibility of the size or shape of a product became a quality issue.

Quality, particularly the dimensions of component parts, became a very serious issue because no longer were the parts hand-built and individually fitted until the product worked. Now, the mass-produced part had to function properly in every product built. Quality was obtained by inspecting each part and passing only those that met specifications. This was true until 1931 when Walter Shewhart, a statistician at the Hawthorne plant at Western Electric, published his book *Economic Control of Quality of Manufactured Product* (Van Nostrand, 1931). This book is the foundation of modern statistical process control (SPC) and provides the basis for the philosophy of total quality management or continuous process improvement for improving processes. With statistical process control, the process is monitored through sampling. Considering the results of the sample, adjustments are made to the process before the process is able to produce defective parts.

Processes and Process Variability

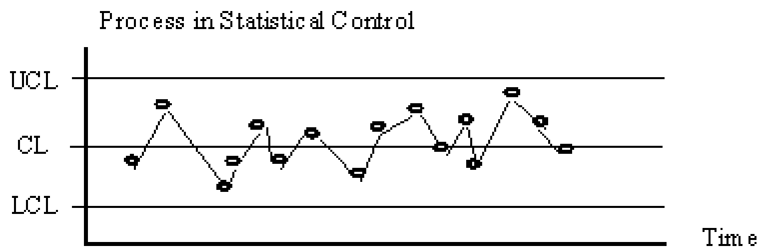
The concept of process variability forms the heart of statistical process control. For example, if a basketball player shot free throws in practice, and the player shot 100 free throws every day, the player would not get exactly the same number of baskets each day. Some days the player would get 84 of 100, some days 67 of 100, some days 77 of 100, and so on. All processes have this kind of variation or variability.

This process variation can be partitioned into two components. Natural process variation, frequently called common cause or system variation, is the naturally occurring fluctuation or variation inherent in all processes. In the case of the basketball player, this variation would fluctuate around the player's long-run percentage of free throws made. Special cause variation is typically caused by some problem or extraordinary occurrence in the system. In the case of the basketball player, a hand injury might cause the player to miss a larger than usual number of free throws on a particular day.

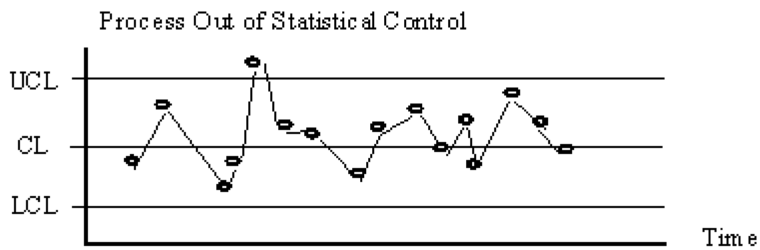
Statistical Process Control

Shewhart's discovery statistical process control or SPC, is a methodology for charting the process and quickly determining when a process is "out of control" (e.g., a special cause variation is present because something unusual is occurring in the process). The process is then investigated to determine the root cause of the "out of control" condition. When the root cause of the problem is determined, a strategy is identified to correct it. The investigation and subsequent correction strategy is frequently a team process and one or more of the TQM process improvement tools are used to identify the root cause. Hence, the emphasis on teamwork and training in process improvement methodology.

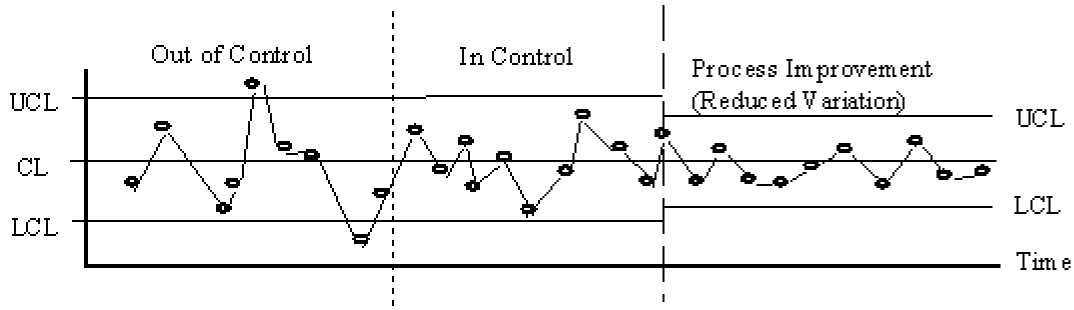
It is management's responsibility to reduce common cause or system variation as well as special cause variation. This is done through process improvement techniques, investing in new technology, or reengineering the process to have fewer steps and therefore less variation. Management wants as little total variation in a process as possible--both common cause and special cause variation. Reduced variation makes the process more predictable with process output closer to the desired or nominal value. The desire for absolutely minimal variation mandates working toward the goal of reduced process variation.



The process above is in apparent statistical control. Notice that all points lie within the upper control limits (UCL) and the lower control limits (LCL). This process exhibits only common cause variation.



The process above is out of statistical control. Notice that a single point can be found outside the control limits (above them). This means that a source of special cause variation is present. The likelihood of this happening by chance is only about 1 in 1,000. This small probability means that when a point is found outside the control limits that it is very likely that a source of special cause variation is present and should be isolated and dealt with. Having a point outside the control limits is the most easily detectable out-of-control condition.



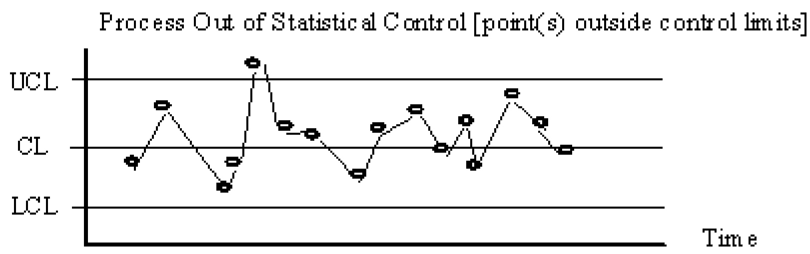
The graphic above illustrates the typical cycle in SPC. First, the process is highly variable and out of statistical control. Second, as special causes of variation are found, the process comes into statistical control. Finally, through process improvement, variation is reduced. This is seen from the narrowing of the control limits. Eliminating special cause variation keeps the process in control; process improvement reduces the process variation and moves the control limits in toward the centerline of the process.

Types of Out-of-Control Conditions

Several types of conditions exist that indicate that a process is out of control. The first of these we have seen already—having one or more points outside the ± 3σ limits as shown below:

Extreme Point Condition

This process is out of control because a point is either above the UCL or below the UCL.

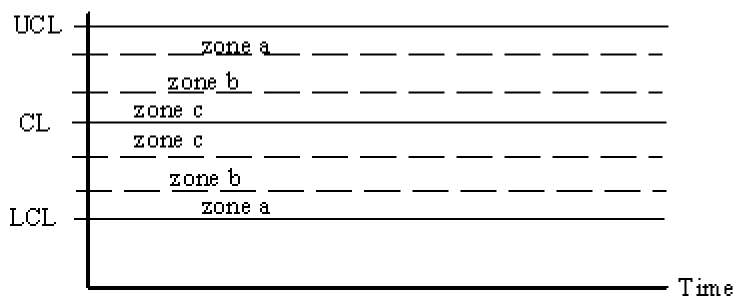


This is the most frequent and obvious out of control condition and is true for all control charts.

Control Chart Zones

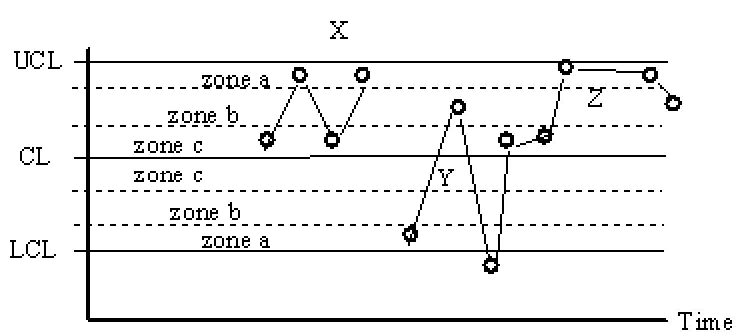
Control charts can be broken into three zones, a, b, and c on each side of the process center line.

A series of rules exist that are used to detect conditions in which the process is behaving abnormally to the extent that an out of control condition is declared.



Two of Three Consecutive Points in Zone A or Outside Zone A

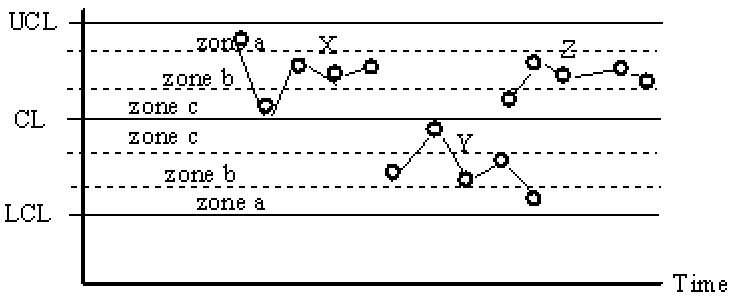
The probability of having two out of three consecutive points either in or beyond zone A is an extremely unlikely occurrence when the process mean follows the normal distribution. Thus, this criteria applies only to $\bar{x}$ charts for examining the process mean.



X, Y, and Z are all examples of this phenomena.

Four of Five Consecutive Points in Zone B or Beyond

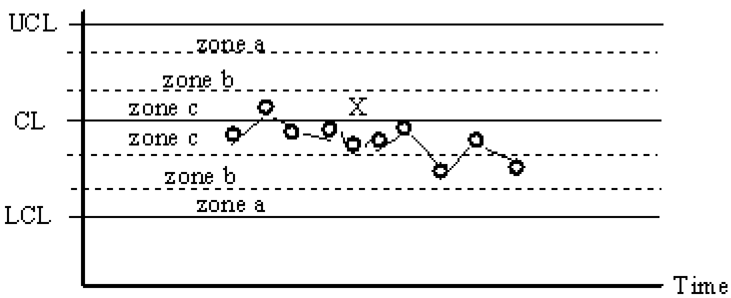
The probability of having four out of five consecutive points either in or beyond zone B is also an extremely unlikely occurrence when the process mean follows the normal distribution. Again this criteria should only be applied to an $\bar{x}$ chart when analyzing a process mean.



X, Y, and Z are all examples of this phenomena.

Runs Above or Below the Centerline

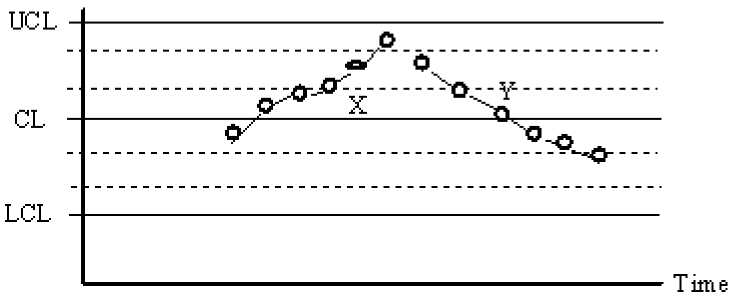
The probability of having long runs (8 or more consecutive points) either above or below the centerline is also an extremely unlikely occurrence when the process follows the normal distribution. This criteria can be applied to both $\bar{x}$ and r charts.



Example X above shows a run below the center line.

Linear Trends

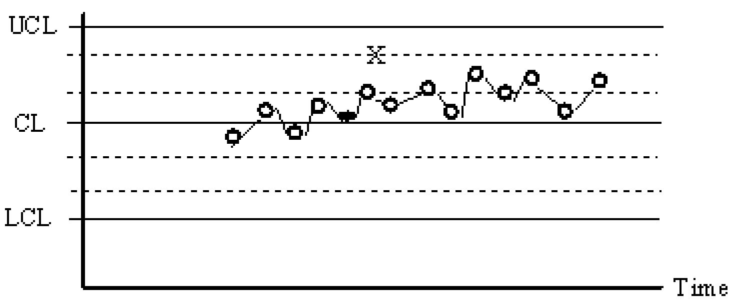
The probability of 6 or more consecutive points showing a continuous increase or decrease is also an extremely unlikely occurrence when the process follows the normal distribution. This criteria can be applied to both $\bar{x}$ and r charts.



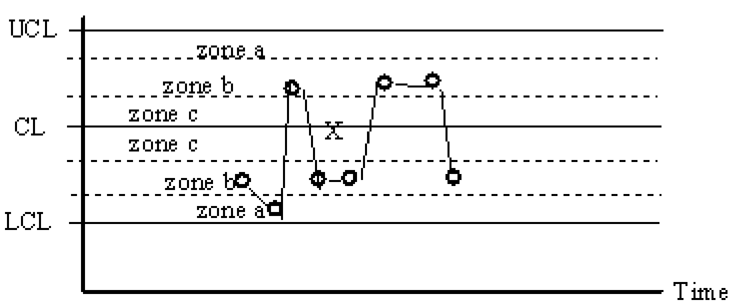
X and Y are both examples of trends. Note that the zones play no part in the interpretation of this out of control condition.

Oscillatory Trend

The probability of having 14 or more consecutive points oscillating back and forth is also an extremely unlikely occurrence when the process follows the normal distribution. It also signals an out of control condition. This criteria can be applied to both $\bar{x}$ and r charts.



X is an example of this out of control condition. Note that the zones play no part in the interpretation of this out of control condition.



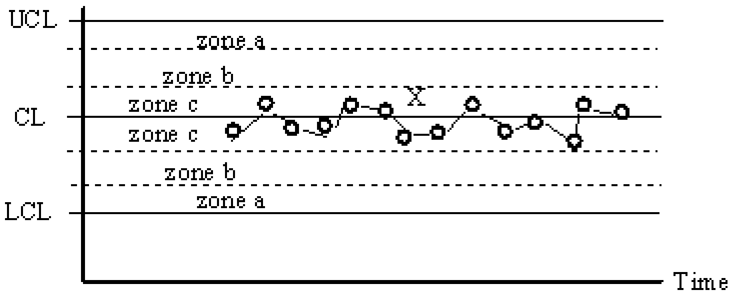
Avoidance of Zone C

The probability of having 8 or more consecutive points occurring on either side of the center line and do not enter Zone C is also an extremely unlikely occurrence when the process follows the normal distribution and signals an out of control condition. This criteria can be applied to $\bar{x}$ charts only. This phenomena occurs when more than one process is being charted on the same chart (probably by accident—e.g., samples from two machines mixed and put on a single chart), the use of improper sampling techniques, or perhaps the process is over controlled or the data is being falsified by someone in the system.

X is an example of this out of control condition.

Run in Zone C

The probability of having 15 or more consecutive points occurring the Zone C is also an extremely unlikely occurrence when the process follows the normal distribution and signals an out of control condition. This criteria can be applied to $\bar{X}$ charts only. This condition can arise from improper sampling, falsification of data, or a decrease in process variability that has not been accounted for when calculating control chart limits, UCL and LCL.



X is an example of this out of control condition.

Sampling and Control Charts

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Several issues are important when selecting sample for control chart purposes. They include:

- sample (subgroup) size considerations
- sampling frequency
- collecting samples

A major goal when selecting a sample from a process is to select the sample (subgroup) in such a way that the variation within the subgroup is attributable only to the random variation inherent in the process or common cause variation. The idea is that the sample should be chosen in such a manner that the chances are maximized to have each element in the sample be alike and subject only to common cause variation. The *spacing* of the samples (subgroups) is arranged so that *if* special cause variation is present, the control chart can identify its presence.

Considerations in Determining Subgroup (Sample) Size

- Subgroups should be small enough to be economically and practically feasible. The time and effort it takes to collect and measure samples weighs heavily here.
- Subgroups should be large enough to allow the central limit theorem to cause the distribution of sample means to be normally distributed. I many cases the process measurements are normal, or close to normal. In a few cases they may not be normal. We know from the central limit theorem that the larger the sample size, the more likely it is that the distribution of sample means will follow the normal distribution. From a practical perspective, this is true for most subgroup sizes of 4 or more.
- Larger subgroups also are needed to provide good sensitivity in detecting out of control conditions. The larger the subgroup size, the more likely it is that a shift in a process mean would be detected.
- As mentioned previously, subgroups should be selected so that they are subject ONLY to common cause variation. If subgroups are allowed to get very large, it is possible that special cause variation can be mixed with common cause. This effect will reduce the sensitivity of the control chart in detecting shifts in the process characteristic of interest.

When all of the above considerations is taken into account, typically a subgroup size of between 4 and 6 is selected. Five is the most commonly used subgroup size (this is due to the historic fact that since 5 is half of 10, many computations with 5 can be done mentally--with calculators and computers, this is probably no longer an important consideration.

Considerations in Collecting Samples

Typically, we want measurement within a subgroup to be taken as close to the same time as possible to reduce the change that special cause variation is present within the subgroup. Thus it is common that consecutive samples from a process are taken. A period of time elapses, and another subgroup sample is collected consecutively.

The spacing between the subgroups shouldn't be exactly uniform. It is not a good idea to take samples EXACTLY every hour or at EXACTLY the same time each day. A certain amount of randomness in the interval between samples is good because it tends to minimize the effect of shift changes, tool wear, tool changes, etc. If the rule is to take samples hourly, a better plan might be to take them hourly, but vary the time randomly within ± 10 minutes of the hour interval.

The Frequency of Sample Collecting

The bottom line is that samples must be collected frequently enough to be useful in identifying and solving problems. In many cases in industry, samples are collected too infrequently. The following should be considered:

- **Process Stability**--If a process has not been analyzed using control charts before and exhibits erratic behavior, sampling should be more frequent to increase the opportunities for process improvement. In this case, frequently ALL parts are sampled, measured, and grouped serially into groups of 5, for example, and then charted. The frequency between these samples of 5 is reduced as the process becomes more stable.
- **Frequency of Process Events**--If a process has may things happening in it, material changes, tool changes, process adjustments, etc., sampling should take place after these potential special causes so they can be detected. When many special events are taking place in a process, each shift, taking two samples (subgroups) per shift will be of little benefit.
- **Sampling Cost**--Two considerations can occur. The time involved in taking the sample is one factor and if the quality characteristic can be observed only through destructive testing, the loss or output can be a significant cost. These costs must be weighed when determining the frequency of sampling. A much more usual condition is that the sampling cost is deemed too high and the frequency of sampling is reduced to the level that the charts derived have no value. In this case, a great deal is spent on sampling with no value derived from the charts and ALL of the expenditures are wasted. Thus, if the process is to be sampled, the samples should be taken frequently enough that the resulting charts are of value. Else, charting these processes should simply be abandoned.

The Problems of Stratification and Mixing

Stratification occurs when the output of several parallel (and assumed identical) processes into a single sample for charting the combined process. Typically a single sample is taken from each machine and included in the subgroup. If a problem develops in the process for one of the machines, it is very difficult to detect because the sample from the "problem" machine is grouped with other samples from "normal" machines. What is plotted as common cause variation is really common cause variation plus the slight differences between the process means of the individual machines. Typically, stratification is detected when large numbers of points lie in Zone C of an $\bar{\bar{x}}$ chart. It looks like the process is in super control when, in fact, the control limits are just calculated too wide. The solution to stratification, obviously, is to chart each machine separately. Control charts are applicable to one and only one process at a time.

Mixing is similar to stratification, except the output of several parallel machines is mixed and the periodic sample is drawn from the mixture. Similar to stratification, mixing will mask problems in individual machines and will make isolation of the problem difficult. Mixing tends to produce an appearance on the control chart where points tend to lie near the control limits that they really should be. The more dissimilar the machines, the more pronounced this phenomina will be. Frequently mixtures come from processes such as multispindle screw machines, multicavity molds. The solution to mixing, obviously, is to chart each machine or mold separately. Control charts are applicable to one and only one process at a time.

Process Capability Analysis

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The capability of a process is defined as the inherent variability of a process in the absence of any undesirable special causes; the smallest variability of which the process is capable with variability due solely to common causes.

Typically, processes follow the normal probability distribution. When this is true, a high percentage of the process measurements fall between $\pm 3\sigma$ of the process mean or center. That is, approximately .27% of the measurements would naturally fall outside the $\pm 3\sigma$ limits and the balance of them (approximately 99.73%) would be within the $\pm 3\sigma$ limits.

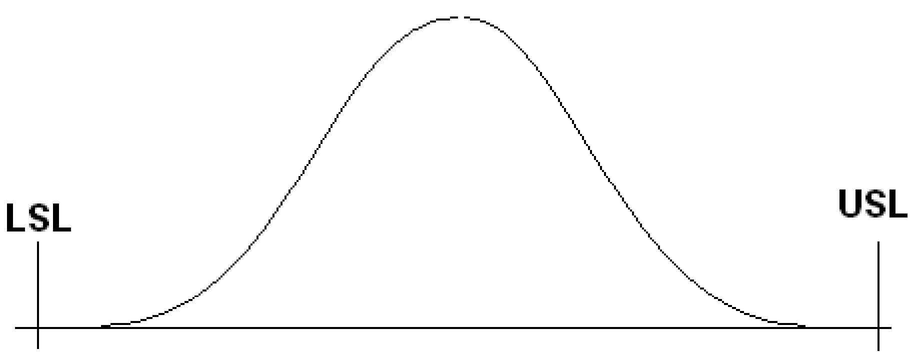
Since the process limits extend from -3σ to $+3\sigma$, the total spread amounts to about 6σ total variation. If we compare process spread with specification spread, we typically have one of three situations:

Case I

A Highly Capable Process

The Process Spread is Well Within the Specification Spread

6σ <[USL-LSL]



When processes are capable, we have an attractive situation for several reasons:

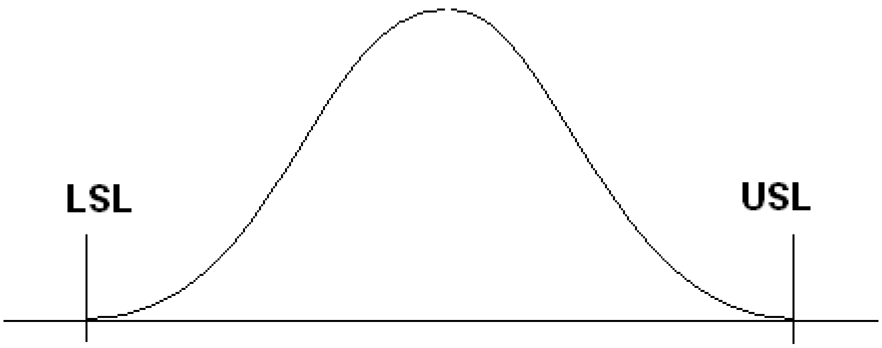
We could tighten our specification limits and claim our product is more uniform or consistent than our competitors. We can rightfully claim that the customer should experience less difficulty, less rework, more reliability, etc. This should translate into higher profits.

Case II

A Barely Capable Process

The Process Spread Just About Matches 6σ

6σ =[USL-LSL]



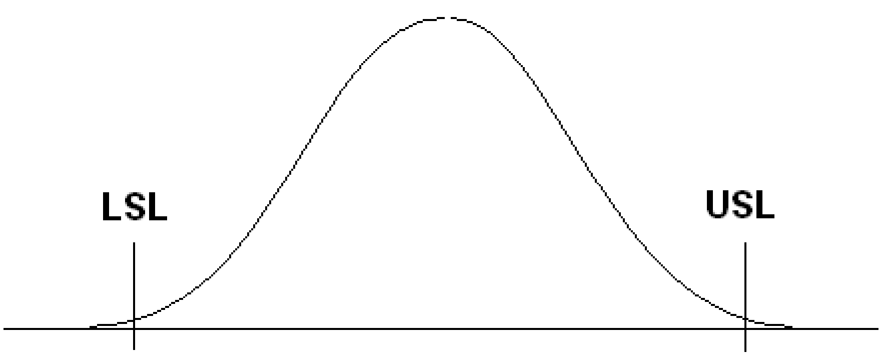
When a process spread is just about equal to the specification spread, the process is capable of meeting specifications, but barely so. This suggests that if the process mean moves to the right or to the left just a little bit, a significant amount of the output will exceed one of the specification limits. The process must be watched closely to detect shifts from the mean. Control charts are excellent tools to do this.

Case III

A Not Capable Process

The Process Spread Is Within 6σ

6σ>[USL-LSL]



When the process spread is greater than the specification spread, a process is not capable of meeting specifications regardless of where the process mean or center is located. This is indeed a sorry situation. Frequently this happens, and the people responsible are not even aware of it. Over adjustment of the process is one consequence, resulting in even greater variability. Alternatives include:

- Changing the process to a more reliable technology or studying the process carefully in an attempt to reduce process variability.
- Live with the current process and sort 100% of the output.
- Re-center the process to minimize the total losses outside the spec limits

- Shut down the process and get out of that business.

Steps in Determining Process Capability

Determine if Specifications are Currently Being Met

1. Collect at least 100 random samples from the process
2. Calculate the sample mean and standard deviation to estimate the true mean and standard deviation of the process.
3. Create a frequency distribution for the data and determine if it is close to being normally distributed. If it is continue; if not, get the help of a statistician to transform the data or to find an alternative model.
4. Plot the USL and LSL on the frequency distribution.
5. If part of the histogram is outside the specification limits, consider adjusting the mean to center the process.
6. If the histogram indicates that the process spread is greater than the specification spread, the process might not be capable.

Determine the Inherent Variability Using an r-Chart

1. Get at least 40 rational subgroups of sample size, preferably at least 4 or 5.
2. Calculate the ranges for each subgroup, the average range, rbar, and the control limits for an r-chart. Plot the data.
3. Discard any ranges outside the UCL ONLY if the undesirable special cause is identifiable and can be removed from the process; otherwise include the offending range(s).
4. Recalculate the average range, rbar, and the control limits.
5. Repeat the last two steps until all ranges are in statistical control.
6. Estimate the process standard deviation, σ, with the formula $\sigma = rbar / d_2$.
7. Using the midpoint of the specifications as the process mean, assuming normality, draw a normal curve, and estimate the percentage meeting specifications. This step assumes that the mean of the process can be adjusted or recentered.
8. If the normal curve shows that the specifications can be met, determine what the specifications are not being met. Create an xbar chart from the same subgroup data and hunt for clues. It may be as simple as recentering the process. Perhaps special causes are present that can be removed.
9. If the specifications cannot be met, consider changing the process by improvement, living with it and sorting 100% of the output, centering the mean, or dropping the product totally.
10. Set up a system of xbar-r charts to create future process improvements and process control.

Capability Indices

Capability indices are simplified measures to quickly describe the relationship between the variability of a process and the spread of the specification limits. Like many simplified measures, such as the grades A, B, C, D, and F in school, capability indices do not completely describe what is happening with a process. They are useful when the assumptions for using them are met to compare the capabilities of processes.

The Capability Index - C_p

The equation for the simplest capability index, C_p, is the ratio of the specification spread to the process spread, the latter represented by six standard deviations or 6σ.

$$C_p = \frac{USL - LCL}{6\sigma}$$

C_p assumes that the normal distribution is the correct model for the process. C_p can be highly inaccurate and lead to misleading conclusions about the process when the process data does not follow the normal distribution.

Occasionally the inverse of the capability index C_p, the capability ratio *CR* is used to describe the percentage of the specification spread that is occupied or used by the process spread.

$$CR = CapabilityRatio = \frac{1}{C_p} x100\% = \frac{6\sigma}{USL - LSL} x100\%$$

C_p can be translated directly to the percentage or proportion of nonconforming product outside specifications. When C_p =1.00, approximately .27% of the parts are outside the specification limits (assuming the process is centered on the midpoint between the specification limits) because the specification limits closely match the process UCL and LCL. We say this is about 2700 parts per million (ppm) nonconforming.

When C_p =1.33, approximately .0064% of the parts are outside the specification limits (assuming the process is centered on the midpoint between the specification limits). We say this is about 64 parts per million (ppm) nonconforming. In this case, we would be looking at normal curve areas beyond 1.33x $3\sigma = \pm 4\sigma$ from the center.

When C_p =1.67, approximately .000057% of the parts are outside the specification limits (assuming the process is centered on the midpoint between the specification limits). We say this is about .6 parts per million (ppm) nonconforming. In this case, we would be looking at normal curve areas beyond 1.67x $3\sigma = \pm 5\sigma$ from the center of the normal distribution. Remember that the capability index C_p ignores the mean or target of the process. If the process mean lined up exactly with one of the specification limits, half the output would be nonconforming regardless of what the value of C_p was. Thus, C_p is a measure of potential to meet specification but says little about current *performance* in doing so.

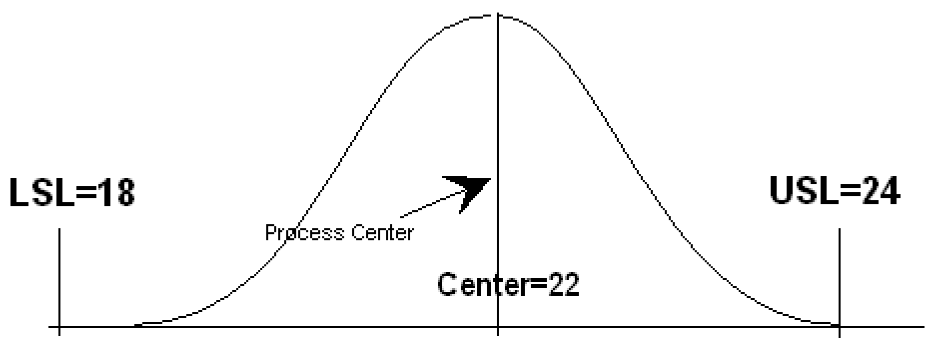
The Capability Index - C_{pk}

The major weakness in C_p was the fact that few, if any processes remain centered on the process mean.

Thus, to get a better measure of the current performance of a process, one must consider where the process mean is located relative to the specification limits. The index C_{pk} was created to do exactly this. With C_{pk}, the location of the process center compared to the USL and LSL is included in the computations and a worst case scenerio is computed in which C_p is computed for the closest specification limit to the process mean.

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma} \, and \, \frac{\mu - LSL}{3\sigma}\right\}$$

We have the following situation. The process standard deviation is σ =.8 with a USL=24, LSL=18, and the process mean μ=22.



$$C_{pk} = \min\left\{\frac{24-22}{3*.8} \, and \, \frac{22-18}{3*.8}\right\}$$ =min{.83 and 1.67}=.83

If this process' mean was exactly centered between the specification limits, $C_p = C_{pk} = 1.25$.

The Capability Index - C_{pm}

C_{pm} is called the Taguchi capability index after the Japanese quality guru, Genichi Taguchi whose work on the Taguchi Loss Function stressed the economic loss incurred as processes departed from target values. This index was developed in the late 1980's and takes into account the proximity of the process mean to a designated target, *T*.

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

When the process mean is centered between the specification limits and the process mean is on the target, T, $C_p = C_{pk} = C_{pm}$.

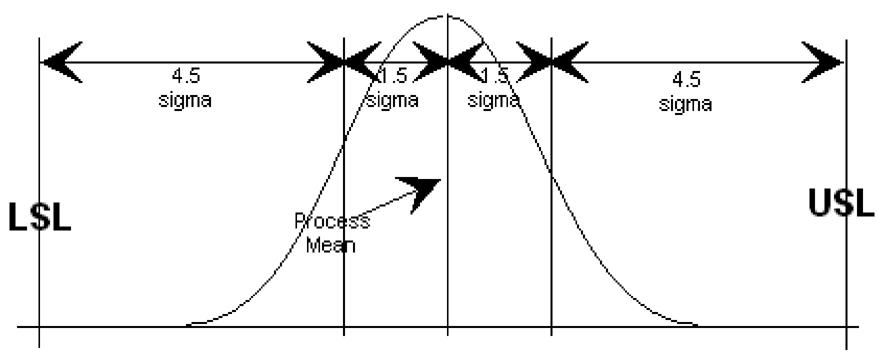
When a process mean departs from the target value T, there is a substantive affect on the capability index. In the C_{pk} example above, if the target value were T=21, C_{pm} would be calculated as:

$$C_{pm} = \frac{24-18}{6\sqrt{.8^2 + (22-21)^2}} = 1.281$$

In this case, the Taguchi capability index is somewhat more liberal than C_{pk}.

Motorola's Six Sigma Quality

In 1988, the Motorola Corporation was the winner of the Malcolm Baldrige National Quality Award. Motorola bases much of its quality effort on what its calls its "6-Sigma" Program. The goal of this program was to reduce the variation in every process to such an extent that a spread of 12σ (6σ on each side of the mean) fits within the process specification limits. Motorola allocates 1.5σ on either side of the process mean for shifting of the mean, leaving 4.5σ between this safety zone and the respective process specification limit.



Thus, even if the process mean strays as much as 1.5σ from the process center, a full 4.5σ remains. This insures a worst case scenerio of 3.4 ppm nonconforming on each side of the distribution (6.8 ppm total) and a best case scenerio of 1 nonconforming part per **billion** (ppb) for each side of the distribution (2 ppb total). If the process mean were centered, this would translate into a C_p=2.00.

Motorola has made significant progress toward this goal across most processes, including many office and business processes as well.

Constructing Run Charts

A run chart is a line graph that shows data points plotted in the order in which they occur. They are used to show trends and shifts in a process over time, variation over time, or to identify decline or improvement in a process over time. They can be used to examine both variables and attribute data.

Steps in Constructing a Run Chart

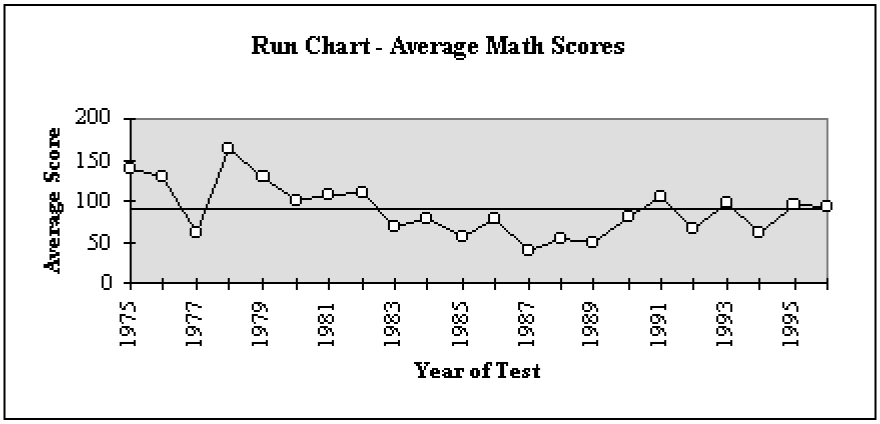
1. Draw and label the vertical (y) axis using the measurement units you are tracking (e.g., numbers of defectives, mean diameter, number of graduates, percent defective, etc.)
2. Draw and label the horizontal (x) axis to reflect the sequence in which the data points are collected (e.g., week 1, week 2, ... or 8AM, 9AM, 10AM, etc.)
3. Plot the data points on the chart in the order in which they became available and connect the points with lines between them.
4. Calculate the average from the data, and draw a horizontal line across the chart at the level of the average.
5. Interpret the chart and decide what action to take. Are trends present? Would the chart look different if everything were perfect? The key is to look for trends, and not focus on individual points.

Example:

Average Math Entrance Exam Scores by Year

Year	Average Math Score
1975	139
1976	130
1977	61
1978	164
1979	129
1980	100
1981	108
1982	110
1983	68
1984	78
1985	57
1986	77
1987	38
1988	53
1989	50
1990	81
1991	105
1992	65
1993	97
1994	62
1995	96
1996	93

Run - Chart:



It should be obvious from this chart, that average math scores in the eighties dropped from those in the seventies, and now, in the nineties have rebounded somewhat but are still lower than they were in the seventies.

XBAR and R Charts

Theoretical Control Limits for XBAR Charts

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}}$$

$$LCL = \mu - \frac{3\sigma}{\sqrt{n}}$$

Although theoretically possible, since we do not know either the population process mean or standard deviation, these formulas cannot be used directly and both must be estimated from the process itself. First the R chart is constructed. If the R chart validates that the process variation is in statistical control, the XBAR chart is constructed.

Steps in Constructing an R Chart

1. Select k successive subgroups where k is at least 20, in which there are n measurements in each subgroup. Typically n is between 1 and 9. 3, 4, or 5 measurements per subgroup is quite common.
2. Find the range of each subgroup R(i) where R(i)=biggest value - smallest value for each subgroup i.
3. Find the centerline for the R chart, denoted by

$$RBAR = \frac{1}{k}\sum R(i)$$

4. Find the UCL and LCL with the following formulas: UCL= D(4)RBAR and LCL=D(3)RBAR with D(3) and D(4) can be found in the following table:

Table of D(3) and D(4)

n	D(3)	D(4)	n	D(3)	D(4)
2	0	3.267	6	0	2.004
3	0	2.574	7	.076	1.924
4	0	2.282	8	.136	1.864
5	0	2.114	9	.184	1.816

5. Plot the subgroup data and determine if the process is in statistical control. If not, determine the reason for the assignable cause, eliminate it, and the subgroup(s) and repeat the previous 3 steps. Do NOT eliminate subgroups with points out of range for which assignable causes cannot be found.
6. Once the R chart is in a state of statistical control and the centerline RBAR can be considered a reliable estimate of the range, the process standard deviation can be estimated using:

$$\hat{\sigma} = \frac{RBAR}{d(2)}$$

d(2) can be found in the following table:

n	d(2)	n	d(2)
2	1.128	6	2.534
3	1.693	7	2.704
4	2.059	8	2.847
5	2.326	9	2.970

Steps in Constructing the XBAR Chart

1. Find the mean of each subgroup XBAR(1), XBAR(2), XBAR(3)... XBAR(k) and the grand mean of all subgroups using:

$$\bar{\bar{x}} = \frac{1}{k}\sum XBAR(i)$$

2. Find the UCL and LCL using the following equations:

$$UCL = \bar{\bar{x}} + A(2)RBAR$$
$$LCL = \bar{\bar{x}} + A(2)RBAR$$

A(2) can be found in the following table:

n	A(2)	n	A(2)
2	1.880	6	.483
3	1.023	7	.419
4	.729	8	.373
5	.577	9	.337

3. Plot the LCL, UCL, centerline, and subgroup means
4. Interpret the data using the following guidelines to determine if the process is in control:

   a. one pount outside the 3 sigma control limits

   b. eight successive points on the same side of the centerline

   c. six successive points that increase or decrease

   d. two out of three points that are on the same side of the centerline, both at a distance exceeding 2 sigmas from the centerline

e. four out of five points that are on the same side of the centerline, four at a distance exceeding 1 sigma from the centerline

f. using an average run length (ARL) for determining process anomolies

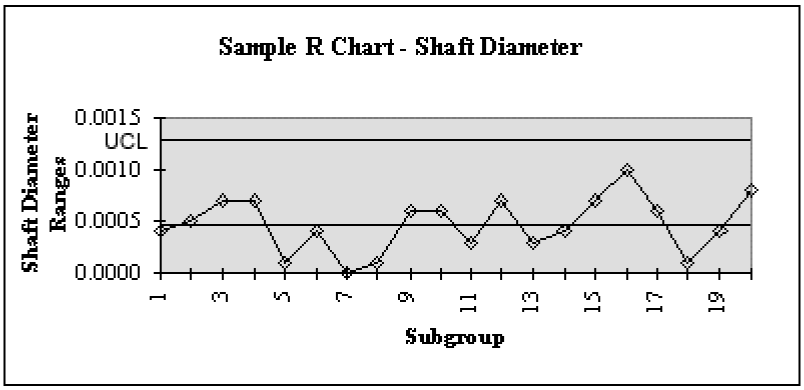
Example:

The following data consists of 20 sets of three measurements of the diameter of an engine shaft.

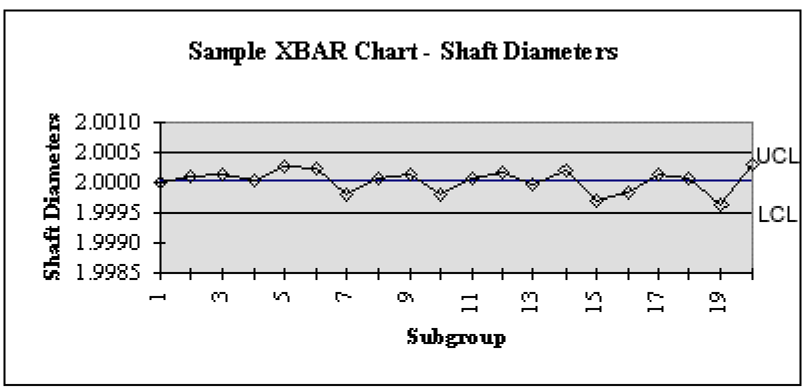
n	meas#1	meas#2	meas#3	Range	XBAR
1	2.0000	1.9998	2.0002	0.0004	2.0000
2	1.9998	2.0003	2.0002	0.0005	2.0001
3	1.9998	2.0001	2.0005	0.0007	2.0001
4	1.9997	2.0000	2.0004	0.0007	2.0000
5	2.0003	2.0003	2.0002	0.0001	2.0003
6	2.0004	2.0003	2.0000	0.0004	2.0002
7	1.9998	1.9998	1.9998	0.0000	1.9998
8	2.0000	2.0001	2.0001	0.0001	2.0001
9	2.0005	2.0000	1.9999	0.0006	2.0001
10	1.9995	1.9998	2.0001	0.0006	1.9998
11	2.0002	1.9999	2.0001	0.0003	2.0001
12	2.0002	1.9998	2.0005	0.0007	2.0002
13	2.0000	2.0001	1.9998	0.0003	2.0000
14	2.0000	2.0002	2.0004	0.0004	2.0002
15	1.9994	2.0001	1.9996	0.0007	1.9997
16	1.9999	2.0003	1.9993	0.0010	1.9998
17	2.0002	1.9998	2.0004	0.0006	2.0001
18	2.0000	2.0001	2.0001	0.0001	2.0001
19	1.9997	1.9994	1.9998	0.0004	1.9996

```
20      2.0003  2.0007  1.9999          0.0008  2.0003


RBAR CHART LIMITS:

RBAR  = 0.0005

UCL=D(4)*RBAR = 2.574 * .0005 = 0.001287
LCL=D(3)*RBAR = 0.000 * .0005 = 0.000


XBAR CHART LIMITS:

XDBLBAR  = 2.0000

UCL  =  XDBLBAR + A(2)*RBAR  =  2.000+1.023*.0005 = 2.0005115
LCL  =  XDBLBAR - A(2)*RBAR  =  2.000-1.023*.0005 = 1.9994885
```

R - Chart:



XBAR - Chart:



XBAR and s Charts

Theoretical Control Limits for XBAR Charts

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}}$$

$$LCL = \mu - \frac{3\sigma}{\sqrt{n}}$$

Although theoretically possible, since we do not know either the population process mean or standard deviation, these formulas cannot be used directly and both must be estimated from the process itself. First the s chart is constructed. If the s chart validates that the process variation is in statistical control, the XBAR chart is constructed.

Steps in Constructing an s Chart

1. Select k successive subgroups where k is at least 20, in which there are n measurements in each subgroup. Typically n is between 1 and 9. 3, 4, or 5 measurements per subgroup is quite common.
2. Find the sample standard deviation of each subgroup s(i).
3. Find the centerline for the s chart, denoted by

$$sbar = \frac{1}{k}\sum s(i)$$

4. Find the UCL and LCL with the following formulas: UCL= B(4)SBAR and LCL=B(3)SBAR with B(3) and B(4) can be found in the following table:

Table of B(3) and B(4)

n	B(3)	B(4)	n	B(3)	B(4)
2	0	3.267	6	.03	1.970
3	0	2.568	7	.118	1.882
4	0	2.266	8	.185	1.815
5	0	2.089	9	.239	1.761

5. Plot the subgroup data and determine if the process is in statistical control. If not, determine the reason for the assignable cause, eliminate it, and the subgroup(s) and repeat the previous 3 steps. Do NOT eliminate subgroups with points out of range for which assignable causes cannot be found.
6. Once the s chart is in a state of statistical control and the centerline SBAR can be considered a reliable estimate of the range, the process standard deviation can be estimated using:

$$\hat{\sigma} = \frac{SBAR}{c(4)}\sqrt{1 - c(4)^2}$$

c(4) can be found in the following table:

n	c(4)	n	c(4)
2	.7979	6	.9515
3	.8862	7	.9594
4	.9213	8	.9650
5	.9400	9	.9693

Steps in Constructing the XBAR Chart

1. Find the mean of each subgroup XBAR(1), XBAR(2), XBAR(3)... XBAR(k) and the grand mean of all subgroups using:

$$\bar{\bar{x}} = \frac{1}{k}\sum XBAR(i)$$

2. Find the UCL and LCL using the following equations:

$$UCL = \bar{\bar{x}} + A(3)SBAR$$
$$LCL = \bar{\bar{x}} - A(3)SBAR$$

A(3) can be found in the following table:

n	A(3)	n	A(3)
2	2.659	6	1.287
3	1.954	7	1.182
4	1.628	8	1.099
5	1.427	9	1.032

3. Plot the LCL, UCL, centerline, and subgroup means
4. Interpret the data using the following guidelines to determine if the process is in control:

```
a.  one pount outside the 3 sigma control limits

b.  eight successive points on the same side of the centerline

c.  six successive points that increase or decrease

d.  two out of three points that are on the same side of the centerline,

    both at a distance exceeding 2 sigmas from the centerline

e.  four out of five points that are on the same side of the centerline,
```

```
     four at a distance exceeding 1 sigma from the centerline

 f.  using an average run length (ARL) for determining process anomolies
```

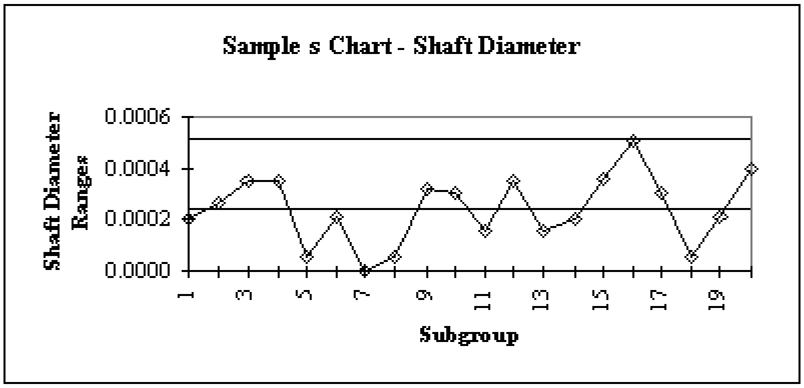
Example:

The following data consists of 20 sets of three measurements of the diameter of an engine shaft.

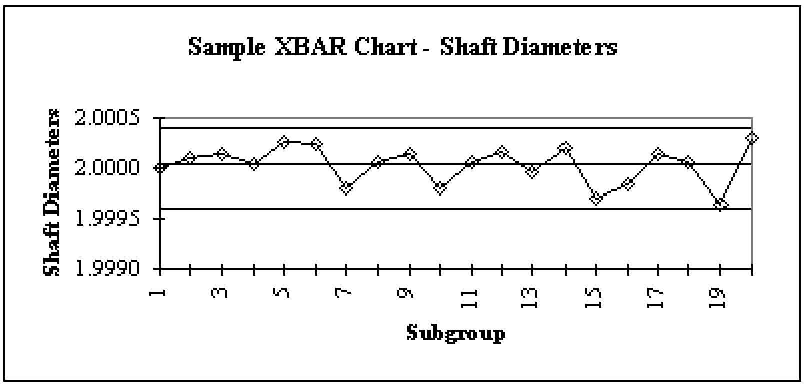
n	meas#1	meas#2	meas#3	StdDev	XBAR
1	2.0000	1.9998	2.0002	0.0002	2.0000
2	1.9998	2.0003	2.0002	0.0003	2.0001
3	1.9998	2.0001	2.0005	0.0004	2.0001
4	1.9997	2.0000	2.0004	0.0004	2.0000
5	2.0003	2.0003	2.0002	0.0001	2.0003
6	2.0004	2.0003	2.0000	0.0002	2.0002
7	1.9998	1.9998	1.9998	0.0000	1.9998
8	2.0000	2.0001	2.0001	0.0001	2.0001
9	2.0005	2.0000	1.9999	0.0003	2.0001
10	1.9995	1.9998	2.0001	0.0003	1.9998
11	2.0002	1.9999	2.0001	0.0002	2.0001
12	2.0002	1.9998	2.0005	0.0004	2.0002
13	2.0000	2.0001	1.9998	0.0002	2.0000
14	2.0000	2.0002	2.0004	0.0002	2.0002
15	1.9994	2.0001	1.9996	0.0004	1.9997
16	1.9999	2.0003	1.9993	0.0005	1.9998
17	2.0002	1.9998	2.0004	0.0003	2.0001
18	2.0000	2.0001	2.0001	0.0001	2.0001
19	1.9997	1.9994	1.9998	0.0002	1.9996
20	2.0003	2.0007	1.9999	0.0004	2.0003

```
SBAR CHART LIMITS:
SBAR  = 0.0002

UCL   =  B(4)*SBAR   =  2.568*.0002 =   0.0005136
LCL   =  B(3)*SBAR   =    0 * .0002 =   0.00

XBAR CHART LIMITS:
XDBLBAR  =              2.0000

UCL  =  XDBLBAR + A(3)*SBAR   = 2.000+1.954*.0002  = 2.0003908
LCL   = XDBLBAR - A(3)*SBAR   = 2.000-1.954*.0002  = 1.9996092
```

s - Chart:


Sample s Chart - Shaft Diameter
Shaft Diameter Ranges
0.0006
0.0004
0.0002
0.0000
1 3 5 7 9 11 13 15 17 19
Subgroup

XBAR - Chart:


Sample XBAR Chart - Shaft Diameters
Shaft Diameters
2.0005
2.0000
1.9995
1.9990
1 3 5 7 9 11 13 15 17 19
Subgroup

Median Charts

Preparing Median Charts

The primary reason for using medians is that it is easier to do on the shop floor because no arithmetic must be done. The person doing the charting can simply order the data and pick the center element. For simplicity, odd numbers of samples are chosen 3, 5, 7, etc. The major disadvantage of using a median chart is that it is less sensitive (powerful) in detecting process changes when extreme values occur.

Traditionally, all subgroup values are plotted, and only the median values are connected by line segments. One must be careful when interpreting the chart that the "out of control" rules are only applied to the median elements.

Steps in Constructing a Median Chart

1. Either an R chart or s chart is developed as shown on the respective XBAR-r or XBAR-s charts and the process variation is shown to be in statistical control.
2. If an R chart was used, the control limits are as follows:

$$UCL = XDBLBAR + A(6) * RBAR$$
$$LCL = XDBLBAR - A(6) * RBAR$$

3. If an s chart was used, the control limits are as follows:

$$UCL = XDBLBAR + A(7) * SBAR$$
$$LCL = XDBLBAR - A(7) * SBAR$$

Table of A(6) and A(7)

n	A(6)	A(7)	n	A(6)	A(7)
2	1.880	1.880	6	.549	.580
3	1.187	1.067	7	.509	.521
4	.796	.796	8	.434	.477
5	.691	.660	9	.412	.444

The centerline is XDBLBAR. Note that it is the subgroup MEANS that determine both the centerline and the control limits.

4. Plot the centerline XDBLBAR, LCL, UCL, and the subgroup medians.
5. Interpret the data using the following guidelines to determine if the process is in control:

a. one pount outside the 3 sigma control limits

b. eight successive points on the same side of the centerline

c. six successive points that increase or decrease

d. two out of three points that are on the same side of the centerline, both at a distance exceeding 2 sigmas from the centerline

e. four out of five points that are on the same side of the centerline,

four at a distance exceeding 1 sigma from the centerline

f. using an average run length (ARL) for determining process anomolies

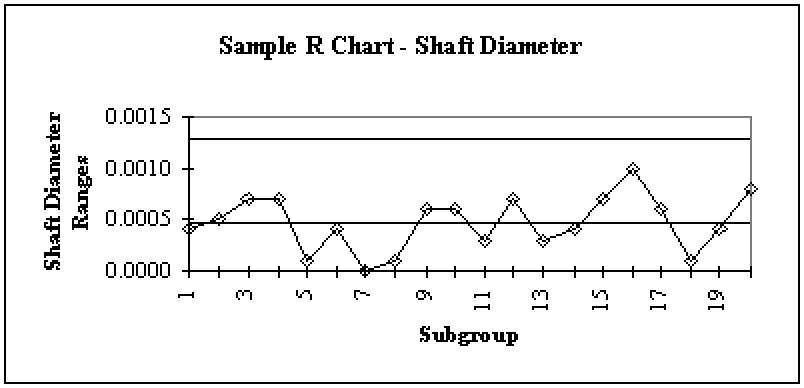
Example:

The following data consists of 20 sets of three measurements of the diameter of an engine shaft. An R-Chart will be used to examine variability followed by a Median Chart.

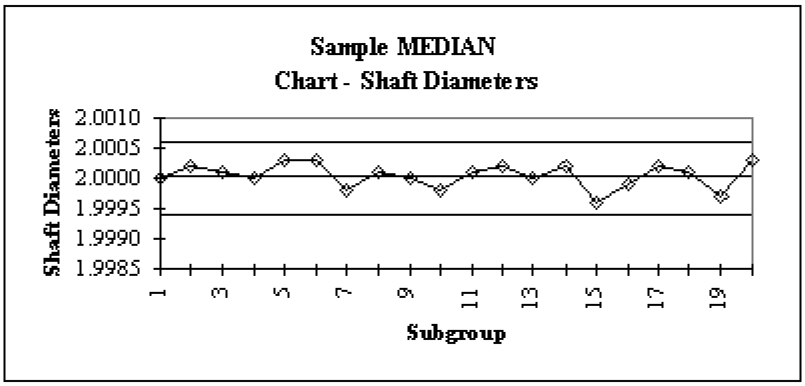
n	meas#1	meas#2	meas#3	Range	XBAR	Median
1	2.0000	1.9998	2.0002	0.0004	2.0000	2.0000
2	1.9998	2.0003	2.0002	0.0005	2.0001	2.0002
3	1.9998	2.0001	2.0005	0.0007	2.0001	2.0001
4	1.9997	2.0000	2.0004	0.0007	2.0000	2.0000
5	2.0003	2.0003	2.0002	0.0001	2.0003	2.0003
6	2.0004	2.0003	2.0000	0.0004	2.0002	2.0003
7	1.9998	1.9998	1.9998	0.0000	1.9998	1.9998
8	2.0000	2.0001	2.0001	0.0001	2.0001	2.0001
9	2.0005	2.0000	1.9999	0.0006	2.0001	2.0000
10	1.9995	1.9998	2.0001	0.0006	1.9998	1.9998
11	2.0002	1.9999	2.0001	0.0003	2.0001	2.0001
12	2.0002	1.9998	2.0005	0.0007	2.0002	2.0002
13	2.0000	2.0001	1.9998	0.0003	2.0000	2.0000
14	2.0000	2.0002	2.0004	0.0004	2.0002	2.0002
15	1.9994	2.0001	1.9996	0.0007	1.9997	1.9996
16	1.9999	2.0003	1.9993	0.0010	1.9998	1.9999
17	2.0002	1.9998	2.0004	0.0006	2.0001	2.0002
18	2.0000	2.0001	2.0001	0.0001	2.0001	2.0001
19	1.9997	1.9994	1.9998	0.0004	1.9996	1.9997
20	2.0003	2.0007	1.9999	0.0008	2.0003	2.0003

```
RBAR CHART LIMITS:
RBAR  = 0.0005

UCL   =  D(4)*RBAR   =  2.574*.0005 =   0.001287
LCL   =  D(3)*RBAR   =  0 * .0005   =   0.00

XBAR CHART LIMITS:
XDBLBAR  = 2.0000

UCL  =  XDBLBAR + A(6)*RBAR   = 2.000+1.187*.0005  = 2.0005935
LCL   = XDBLBAR - A(6)*RBAR   = 2.000-1.187*.0005  = 1.9994065
```

R Chart:



Median - Chart:



Individuals Charts

Preparing Individuals Charts

An Individuals Chart is used when the nature of the process is such that it is difficult or impossible to group measurements into subgroups so an estimate of the process variation can be determined. This occurs frequently in low volume production situations and in situations in chich continuously varying quantities within the process are process-related variables.

The solution is to artificially create subgroups from the data and then calculate the range of each subgroup. This is done by creating rolling groups (most often pairs) of data through time and using the pairs to determine the range R. The resulting ranges are called *moving ranges*.

Steps in Constructing an Individuals Chart

1. A *moving range average* is calculated by taking pairs of data (x1,x2), (x2,x3), (x3,x4),..., (xn-1,xn), taking the sum of the absolute value of the differences between them and dividing by the number of pairs (one less than the number of pieces of data). This is shown mathematically as:

$$MRBAR = \frac{1}{n-1}\sum_{i=1}^{n-1}\left|x_{i+1} - x_i\right|$$

2. An estimate of the process standard deviation is given by

$$\hat{\sigma} = \frac{MRBAR}{1.128}$$

   and the three sigma control limits become:

$$UCL = XBAR + 2.66 * MRBAR$$
$$LCL = XBAR - 2.66 * MRBAR$$

3. Plot the centerline, XBAR, LCL, UCL, and the process measurement X(i).
4. Interpret the data using the following guidelines to determine if the process is in control:

```
a.  one pount outside the 3 sigma control limits

b.  eight successive points on the same side of the centerline

c.  six successive points that increase or decrease

d.  two out of three points that are on the same side of the centerline,

    both at a distance exceeding 2 sigmas from the centerline

e.  four out of five points that are on the same side of the centerline,

    four at a distance exceeding 1 sigma from the centerline
```

f. using an average run length (ARL) for determining process anomolies

Some Additional Notes:

If the process has been brought into statistical control, the sample standard deviation, s/c(4), can well be used as an estimate of the process standard deviation sigma. This works well unless there are trends in the data (which cause an inflated value of s). When one is certain that no trends exist in the data, s/c(4), will provide considerable more power than MRBAR/1.128. This means that one is more likely to detect out-of-control situations when they exist.

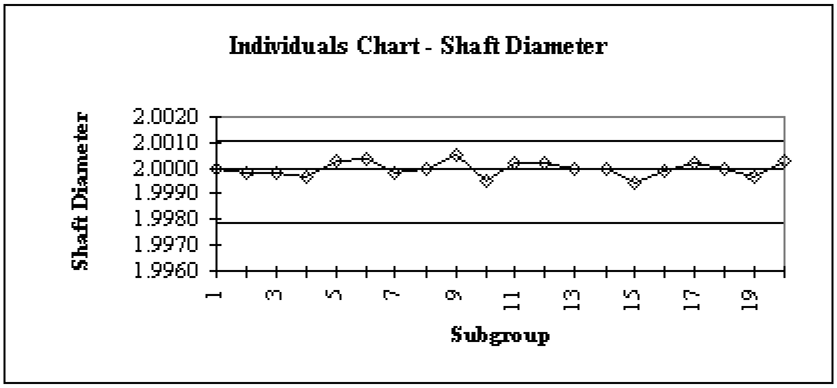
Example:

The following data consists of 20 sets of three measurements of the diameter of an engine shaft. An R-Chart will be used to examine variability followed by a Median Chart.

n	meas#1	MR
1	2.0000	0.0002
2	1.9998	0.0000
3	1.9998	0.0001
4	1.9997	0.0006
5	2.0003	0.0001
6	2.0004	0.0006
7	1.9998	0.0002
8	2.0000	0.0005
9	2.0005	0.0010
10	1.9995	0.0007
11	2.0002	0.0000
12	2.0002	0.0002
13	2.0000	0.0000
14	2.0000	0.0006
15	1.9994	0.0005
16	1.9999	0.0003
17	2.0002	0.0002
18	2.0000	0.0003
19	1.9997	0.0006
20	2.0003	

```
INDIVIDUALS CHART LIMITS:

XBAR =  2.0000
MRBAR  =        0.0004

UCL  =  XBAR + 2.66*MRBAR   =   2.000+2.66*.0004  =     2.001064
LCL   = XBAR - 2.66*MRBAR   =   2.000-2.66*.0004  =     1.997872
```

Individuals Chart:


Individuals Chart - Shaft Diameter
Shaft Diameter
2.0020
2.0010
2.0000
1.9990
1.9980
1.9970
1.9960
1
3
5
7
9
11
13
15
17
19
Subgroup

Exponentially Weighted Moving Average (EWMA) Charts

An EWMA *(Exponentially Weighted Moving Average)* Chart is used when it is desirable to detect out-of-control situations very quickly. EWMA charts have a built in mechanism for incorporating information from all previous subgroups, weighting the information from the closest subgroup with a higher weight. Thus, the control/out-of-control decision is made with information from previous subgroups as well as the current subgroup. The chief advantage of EWMA charts is that they detect out-of-control conditions more quickly than XBAR charts and that this detection can be done by using only one rule...being within or outside the 3-sigma limits. The chief disadvantage is the EWMA chart is that it is more difficult to construct. When the subgroup size is n>1, the EWMA chart is an alternative to an XBAR chart; when the subgroup size is n=1, the EWMA chart is an alternative to the Individuals Chart.

Steps in Constructing EWMA Charts

1. Estimate the process standard deviation, sigma, using RBAR/d(2) or SBAR/c(4) if n1 or by using MRBAR/1.128 if n=1.
2. Determine a weighting constant, lambda, that weights past and current information. If, for example, lambda=.3, 70% of the weight will be given to past information and 30% to current information. Typically a lambda between .1 and .3 provides a reasonable balance between past and current information and .2 is very common in actual practice.
3. Determine the points on the EWMA chart denoted by xhat(1), xhat(2),...,xhat(k)k and computed by using the equation xhat(i)=lambda*xhat(i)+(1-lambda)*xhat(i-1). This recursive formula begins by using as its initial value xhat(0)= XDBLBAR.
4. The control limits are not straight lines in the early stages of the chart. The UCL increases and then stabilizes a fixed distance above the centerline while the LCL decreases and then stabilizes a fixed distance below the centerline.

$$UCL = XHAT(0) + \frac{3 \cdot SIGMA}{\sqrt{n}} \sqrt{\left(\frac{lambda}{1 - lambda}\right)(1 - (1 - lambda)^{2i})}$$

$$UCL = XHAT(0) - \frac{3 \cdot SIGMA}{\sqrt{n}} \sqrt{\left(\frac{lambda}{1 - lambda}\right)(1 - (1 - lambda)^{2i})}$$

5. Plot the centerline, XDBLBAR, the LCL and UCL, and the process measurements XHAT(i).
6. Interpret the data. The chart is out of control only if a point is outside the +/- sigma limits.

Example:

The following data consists of 20 sets of three measurements of the diameter of an engine shaft. Lambda has been chosen as .2.

```
Data and Preliminary Computations:
                                                            lambda
n       meas#1  meas#2  meas#3          Range   XBAR    XHAT    0.2
0                                                       2.0000
1       2.0000  1.9998  2.0002          0.0004  2.0000  2.0000
2       1.9998  2.0003  2.0002          0.0005  2.0001  2.0000
3       1.9998  2.0001  2.0005          0.0007  2.0001  2.0001
```

4	1.9997	2.0000	2.0004	0.0007	2.0000	2.0001
5	2.0003	2.0003	2.0002	0.0001	2.0003	2.0001
6	2.0004	2.0003	2.0000	0.0004	2.0002	2.0001
7	1.9998	1.9998	1.9998	0.0000	1.9998	2.0001
8	2.0000	2.0001	2.0001	0.0001	2.0001	2.0001
9	2.0005	2.0000	1.9999	0.0006	2.0001	2.0001
10	1.9995	1.9998	2.0001	0.0006	1.9998	2.0000
11	2.0002	1.9999	2.0001	0.0003	2.0001	2.0000
12	2.0002	1.9998	2.0005	0.0007	2.0002	2.0001
13	2.0000	2.0001	1.9998	0.0003	2.0000	2.0000
14	2.0000	2.0002	2.0004	0.0004	2.0002	2.0001
15	1.9994	2.0001	1.9996	0.0007	1.9997	2.0000
16	1.9999	2.0003	1.9993	0.0010	1.9998	2.0000
17	2.0002	1.9998	2.0004	0.0006	2.0001	2.0000
18	2.0000	2.0001	2.0001	0.0001	2.0001	2.0000
19	1.9997	1.9994	1.9998	0.0004	1.9996	1.9999
20	2.0003	2.0007	1.9999	0.0008	2.0003	2.0000

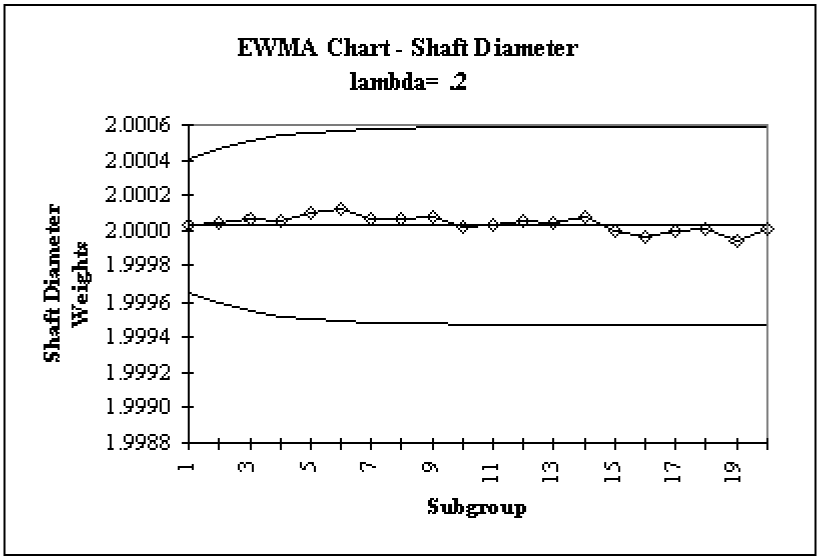
ESTIMATE PROCESS STD DEVIATION:
RBAR = 0.0005
SIGMA = RBAR/d(2) =.0005/1.693 =0.000721501

EWMA CHART LIMITS:
XDBLBAR = 2.0000
UCL=XHAT(0)+((3*SIGMA)/SQRT(n))*SQRT((lambda/(1-lambda))(1-(1-lambda)^(2*i)))
LCL=XHAT(0)-((3*SIGMA)/SQRT(n))*SQRT((lambda/(1-lambda))(1-(1-lambda)^(2*i)))

Control Chart Computations:

Subgrp	CL	LCL	UCL	XHAT
1	2.0000	2.000406569	1.999656764	2.0000
2	2.0000	2.00046109	1.999602243	2.0000
3	2.0000	2.00051173	1.999551604	2.0001
4	2.0000	2.000541506	1.999521827	2.0001
5	2.0000	2.000559683	1.999503651	2.0001
6	2.0000	2.000570994	1.999492339	2.0001
7	2.0000	2.000578111	1.999485223	2.0001
8	2.0000	2.000582617	1.999480716	2.0001
9	2.0000	2.000585482	1.999477851	2.0001
10	2.0000	2.000587308	1.999476026	2.0000
11	2.0000	2.000588473	1.99947486	2.0000
12	2.0000	2.000589218	1.999474116	2.0001
13	2.0000	2.000589694	1.99947364	2.0000
14	2.0000	2.000589998	1.999473335	2.0001
15	2.0000	2.000590193	1.999473141	2.0000
16	2.0000	2.000590317	1.999473016	2.0000
17	2.0000	2.000590397	1.999472936	2.0000
18	2.0000	2.000590448	1.999472885	2.0000
19	2.0000	2.000590481	1.999472853	1.9999
20	2.0000	2.000590502	1.999472832	2.0000

EWMA - Chart:


EWMA Chart - Shaft Diameter
lambda= .2
2.0006
2.0004
2.0002
2.0000
1.9998
1.9996
1.9994
1.9992
1.9990
1.9988
Shaft Diameter Weights
1
3
5
7
9
11
13
15
17
19
Subgroup

Attribute Charts in General, p Charts in Particular

Attribute control charts arise when items are compared with some standard and then are classified as to whether they meet the standard or not. The control chart is used to determine if the rate of nonconforming product is stable and detect when a deviation from stability has occurred. The argument can be made that a LCL should not exist, since rates of nonconforming product outside the LCL is in fact a good thing; we WANT low rates of nonconforming product. However, if we treat these LCL violations as simply another search for an assignable cause, we may learn for the drop in nonconformities rate and be able to permanently improve the process.

p Charts can be used when the subgroups are not of equal size. The np chart is used in the more limited case of equal subgroups.

Steps in Constructing a p Chart

1. Determine the size of the subgroups needed. The size, n(i), has to be sufficiently large to have defects present in the subgroup most of the time. If we have some idea as to what the historical rate of nonconformance, p, is we can use the following formula to estimate the subgroup size:

   n=3/p

2. Determine the rate of nonconformities in each subgroup by using:

   phat(i)=x(i)/n(i)

   where:

   phat(i)=the rate of nonconformities in subgroup i

   x(i)=the number of nonconformities in subgroup i

   n(i)= the size of subgroup i

3. Find pbar; there are k subgroups.

   $$pbar = \frac{1}{k}\sum phat(i)$$

4. Estimate sigma-p if needed and determine the UCL and LCL:

   $$sigma\_p = \sqrt{\frac{pbar \cdot (1 - pbar)}{n}}$$

$$UCL = pbar + 3\sqrt{\frac{pbar(1-pbar)}{n(i)}}$$

$$LCL = pbar - 3\sqrt{\frac{pbar(1-pbar)}{n(i)}}$$

5. Plot the centerline, pbar, the LCL and UCL, and the process measurements, the phat's.
6. Interpret the data to determine if the process is in control.

Example:

Farnum Example:
data is from Farnum (1994):
Modern Statistical Quality Control and Improvement, p. 242

Day	Rejects	Number Tested	Proportion
1	14	286	0.0490
2	22	281	0.0783
3	9	310	0.0290
4	19	313	0.0607
5	21	293	0.0717
6	18	305	0.0590
7	16	322	0.0497
8	16	316	0.0506
9	21	293	0.0717
10	14	287	0.0488
11	15	307	0.0489
12	16	328	0.0488
13	21	296	0.0709
14	9	296	0.0304
15	25	317	0.0789
16	15	297	0.0505
17	14	283	0.0495
18	13	321	0.0405
19	10	317	0.0315
20	21	307	0.0684
21	19	317	0.0599
22	23	323	0.0712
23	15	304	0.0493
24	12	304	0.0395
25	19	324	0.0586
26	17	289	0.0588
27	15	299	0.0502
28	13	318	0.0409
29	19	313	0.0607
30	12	289	0.0415

Calculations:

PBAR = 0.0539

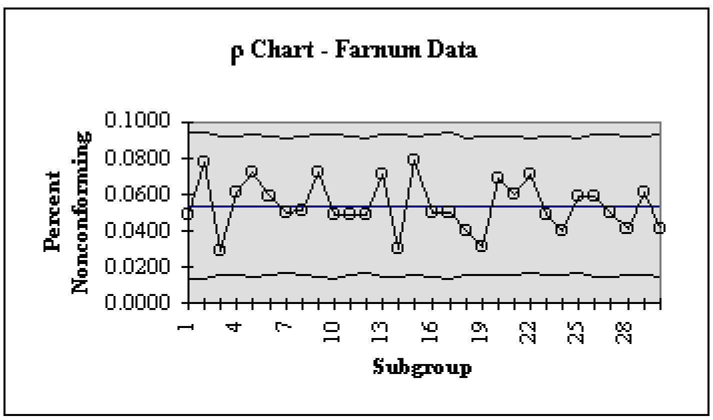
UCL = pbar + 3*sqrt(pbar*(1-pbar)/n(i))

LCL = pbar - 3*sqrt(pbar*(1-pbar)/n(i))

Day CL UCL LCL Proportion

1	0.0539	0.093892049	0.013808661	0.0490
2	0.0539	0.094246721	0.013453989	0.0783
3	0.0539	0.092310827	0.015389883	0.0290
4	0.0539	0.092126068	0.015574642	0.0607
5	0.0539	0.093410843	0.014289867	0.0717
6	0.0539	0.092624795	0.015075915	0.0590
7	0.0539	0.091587368	0.016113342	0.0497
8	0.0539	0.091943946	0.015756764	0.0506
9	0.0539	0.093410843	0.014289867	0.0717
10	0.0539	0.093822229	0.013878481	0.0488
11	0.0539	0.092498288	0.015202422	0.0489
12	0.0539	0.091240619	0.016460091	0.0488
13	0.0539	0.093209857	0.014490853	0.0709
14	0.0539	0.093209857	0.014490853	0.0304
15	0.0539	0.091883814	0.015816896	0.0789
16	0.0539	0.09314354	0.01455717	0.0505
17	0.0539	0.094103724	0.013596986	0.0495
18	0.0539	0.091646103	0.016054607	0.0405
19	0.0539	0.091883814	0.015816896	0.0315
20	0.0539	0.092498288	0.015202422	0.0684
21	0.0539	0.091883814	0.015816896	0.0599
22	0.0539	0.091528906	0.016171804	0.0712
23	0.0539	0.092688517	0.015012193	0.0493
24	0.0539	0.092688517	0.015012193	0.0395
25	0.0539	0.091470715	0.016229995	0.0586
26	0.0539	0.093683678	0.014017032	0.0588
27	0.0539	0.093011904	0.014688806	0.0502
28	0.0539	0.091823966	0.015876744	0.0409
29	0.0539	0.092126068	0.015574642	0.0607
30	0.0539	0.093683678	0.014017032	0.0415

p - Chart:



Attribute Charts in General, np Charts in Particular

Attribute control charts arise when items are compared with some standard and then are classified as to whether they meet the standard or not. The control chart is used to determine if the rate of nonconforming product is stable and detect when a deviation from stability has occurred. The argument can be made that a LCL should not exist, since rates of nonconforming product outside the LCL is in fact a good thing; we WANT low rates of nonconforming product. However, if we treat these LCL violations as simply another search for an assignable cause, we may learn for the drop in nonconformities rate and be able to permanently improve the process.

The np Chart can be used for the special case when the subgroups are of equal size. Then it is not necessary to convert nonconforming counts into the proportions phat(i). Rather, one can directly plot the counts x(i) versus the subgroup number i.

Steps in Constructing an np Chart

1. Determine the size of the subgroups needed. The size, n, has to be sufficiently large to have defects present in the subgroup most of the time. If we have some idea as to what the historical rate of nonconformance, p, is we can use the following formula to estimate the subgroup size:

   n=3/p

2. Find find pbar.

   $$pbar = \frac{x(1) + x(2) + \ldots + x(k)}{k \cdot n}$$

3. Find the UCL and LCL where:

   $$UCL = n \cdot pbar + 3\sqrt{n \cdot pbar(1 - pbar)}$$
   $$LCL = n \cdot pbar - 3\sqrt{n \cdot pbar(1 - pbar)}$$

4. Plot the centerline pbar, the LCL and UCL, and the process nonconforming counts, the x(i)'s.
5. Interpret the control chart. Only if a point is outside the +/- 3 sigma range is the process considered to be out of control.

Example:

```
Farnum Example:
data is from Farnum (1994):
Modern Statistical Quality Control and Improvement, p. 245
```

Day	Non-conforming	Sample Size
1	10	100
2	12	100
3	10	100

4	11	100
5	6	100
6	7	100
7	12	100
8	10	100
9	6	100
10	11	100
11	9	100
12	14	100
13	16	100
14	21	100
15	20	100
16	12	100
17	11	100
18	6	100
19	10	100
20	10	100
21	11	100
22	11	100
23	11	100
24	6	100
25	9	100

Calculations:

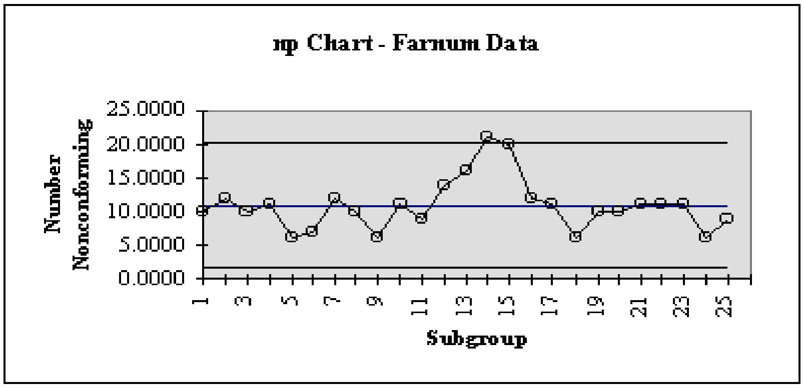
PBAR = 0.1088
CL = 10.8800

UCL = n*pbar + 3*sqrt(n*pbar*(1-pbar))

LCL = n*pbar + 3*sqrt(n*pbar*(1-pbar))

Day	CL	UCL	LCL	NonConforming
1	10.8800	20.22164354	1.538356462	10.0000
2	10.8800	20.22164354	1.538356462	12.0000
3	10.8800	20.22164354	1.538356462	10.0000
4	10.8800	20.22164354	1.538356462	11.0000
5	10.8800	20.22164354	1.538356462	6.0000
6	10.8800	20.22164354	1.538356462	7.0000
7	10.8800	20.22164354	1.538356462	12.0000
8	10.8800	20.22164354	1.538356462	10.0000
9	10.8800	20.22164354	1.538356462	6.0000
10	10.8800	20.22164354	1.538356462	11.0000
11	10.8800	20.22164354	1.538356462	9.0000
12	10.8800	20.22164354	1.538356462	14.0000
13	10.8800	20.22164354	1.538356462	16.0000
14	10.8800	20.22164354	1.538356462	21.0000
15	10.8800	20.22164354	1.538356462	20.0000
16	10.8800	20.22164354	1.538356462	12.0000
17	10.8800	20.22164354	1.538356462	11.0000
18	10.8800	20.22164354	1.538356462	6.0000
19	10.8800	20.22164354	1.538356462	10.0000
20	10.8800	20.22164354	1.538356462	10.0000
21	10.8800	20.22164354	1.538356462	11.0000
22	10.8800	20.22164354	1.538356462	11.0000
23	10.8800	20.22164354	1.538356462	11.0000
24	10.8800	20.22164354	1.538356462	6.0000
25	10.8800	20.22164354	1.538356462	9.0000

np - Chart:


np Chart - Farnum Data
Number Nonconforming
25.0000
20.0000
15.0000
10.0000
5.0000
0.0000
1
3
5
7
9
11
13
15
17
19
21
23
25
Subgroup

Attribute Charts in General, c Charts in Particular

Attribute control charts arise when items are compared with some standard and then are classified as to whether they meet the standard or not. The control chart is used to determine if the rate of nonconforming product is stable and detect when a deviation from stability has occurred. The argument can be made that a LCL should not exist, since rates of nonconforming product outside the LCL is in fact a good thing; we WANT low rates of nonconforming product. However, if we treat these LCL violations as simply another search for an assignable cause, we may learn for the drop in nonconformities rate and be able to permanently improve the process.

The c Chart measures the number of nonconformities per "unit" and is denoted by c. This "unit" is commonly referred to as an inspection unit and may be "per day" or "per square foot" of some other predetermined sensible rate.

Steps in Constructing a c Chart

1. Determine cbar.

   $$cbar = \frac{1}{k}\sum c(i)$$

   There are k inspection units and c(i) is the number of nonconformities in the ith sample.

2. Since the mean and variance of the underlying Poisson distribution are equal,

   $$\hat{\sigma}^2 = cbar$$

   Thus,

   $$\hat{\sigma} = \sqrt{cbar}$$

   and the UCL and LCL are:

   $$UCL = cbar + 3\cdot\sqrt{cbar}$$
   $$LCL = cbar - 3\cdot\sqrt{cbar}$$

3. Plot the centerline cbar, the LCL and UCL, and the process measurements c(i).
4. Interpret the control chart.

Example:

```
Farnum Example:
data is from Farnum (1994):
Modern Statistical Quality Control and Improvement, p. 248

        Non-conforming
```

Day	Errors/1000 lines
1	6
2	7
3	7
4	6
5	8
6	6
7	5
8	8
9	1
10	6
11	2
12	5
13	5
14	4
15	3
16	3
17	2
18	0
19	0
20	1
21	2
22	5
23	1
24	7
25	7
26	1
27	5
28	5
29	8
30	8

Calculations:

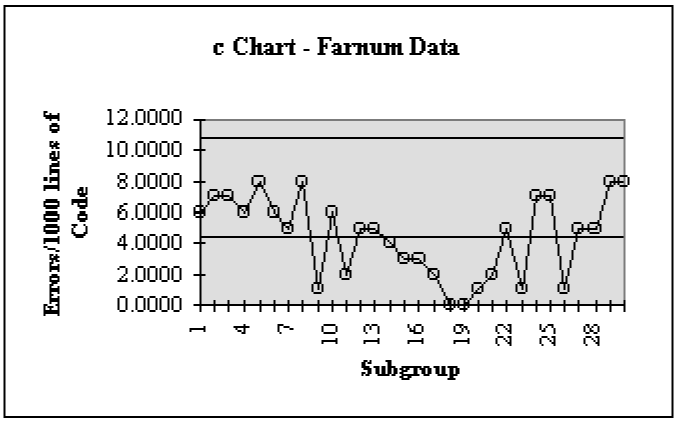
CBAR = 4.4667

UCL = cbar + 3*sqrt(cbar) = 10.80701366
LCL = cbar - 3*sqrt(cbar) = -1.873680327 = 0
(when LCL < 0, set LCL = 0)

Day	CL	UCL	LCL	NonConforming
1	4.4667	10.80701366	0	6
2	4.4667	10.80701366	0	7
3	4.4667	10.80701366	0	7
4	4.4667	10.80701366	0	6
5	4.4667	10.80701366	0	8
6	4.4667	10.80701366	0	6
7	4.4667	10.80701366	0	5
8	4.4667	10.80701366	0	8
9	4.4667	10.80701366	0	1
10	4.4667	10.80701366	0	6
11	4.4667	10.80701366	0	2
12	4.4667	10.80701366	0	5
13	4.4667	10.80701366	0	5
14	4.4667	10.80701366	0	4
15	4.4667	10.80701366	0	3
16	4.4667	10.80701366	0	3
17	4.4667	10.80701366	0	2
18	4.4667	10.80701366	0	0

19	4.4667	10.80701366	0	0
20	4.4667	10.80701366	0	1
21	4.4667	10.80701366	0	2
22	4.4667	10.80701366	0	5
23	4.4667	10.80701366	0	1
24	4.4667	10.80701366	0	7
25	4.4667	10.80701366	0	7
26	4.4667	10.80701366	0	1
27	4.4667	10.80701366	0	5
28	4.4667	10.80701366	0	5
29	4.4667	10.80701366	0	8
30	4.4667	10.80701366	0	8

c - Chart:



Attribute Charts in General, u Charts in Particular

Attribute control charts arise when items are compared with some standard and then are classified as to whether they meet the standard or not. The control chart is used to determine if the rate of nonconforming product is stable and detect when a deviation from stability has occurred. The argument can be made that a LCL should not exist, since rates of nonconforming product outside the LCL is in fact a good thing; we WANT low rates of nonconforming product. However, if we treat these LCL violations as simply another search for an assignable cause, we may learn for the drop in nonconformities rate and be able to permanently improve the process.

The u Chart is used when it is not possible to have an inspection unit of a fixed size (e.g., 12 defects counted in one square foot), rather the number of nonconformities is *per inspection unit* where the inspection unit may not be exactly one square foot...it may be an intact panel or other object, different in sizethan exactly one square foot. When it is converted into a ratio per square foot, or some other measure, it may be controlled with a u chart. Notice that the number no longer has to be integer as with the c chart.

Steps in Constructing a u Chart

1. Find the number of nonconformities, c(i) and the number of inspection units, n(i), in each sample i.
2. Compute u(i)=c(i)/n(i)
3. Determine the centerline of the u chart:

$$ubar = \frac{total\_nonconformities\_in\_k\_subgroups}{total\_number\_of\_inspection\_units}$$

$$ubar = \frac{c(1) + c(2) + \ldots + c(k)}{n(1) + n(2) + \ldots + n(k)}$$

4. The u chart has individual control limits for each subgroup i.

$$UCL = ubar + 3\sqrt{\frac{ubar}{n(i)}}$$

$$UCL = ubar - 3\sqrt{\frac{ubar}{n(i)}}$$

5. Plot the centerline, ubar, the individual LCL's and UCL's, and the process measurements, u(i).
6. Interpret the control chart.

Example:

```
Besterfield Example:
data is from Besterfield (1990): Quality Control  p. 185

                        Number          Nonconformities
```

Day	Number Inspected	Non-Conformities	Per Unit
1	110	120	1.0909
2	82	94	1.1463
3	96	89	0.9271
4	115	162	1.4087
5	108	150	1.3889
6	56	82	1.4643
7	120	143	1.1917
8	98	134	1.3673
9	102	97	0.9510
10	115	145	1.2609
11	88	128	1.4545
12	71	83	1.1690
13	95	120	1.2632
14	103	116	1.1262
15	113	127	1.1239
16	85	92	1.0824
17	101	140	1.3861
18	42	60	1.4286
19	97	121	1.2474
20	92	108	1.1739
21	100	131	1.3100
22	115	119	1.0348
23	99	93	0.9394
24	57	88	1.5439
25	89	107	1.2022
26	101	105	1.0396
27	122	143	1.1721
28	105	132	1.2571
29	98	100	1.0204
30	48	60	1.2500

Calculations:

UBAR = 1.2005

UCL = ubar + 3*sqrt(ubar/n(i))
LCL = ubar - 3*sqrt(ubar/n(i))

Day	CL	UCL	LCL	Nonconformities/Unit
1	1.2005	1.513900448	0.887091405	1.09
2	1.2005	1.563485937	0.837505915	1.15
3	1.2005	1.535975424	0.865016429	0.93
4	1.2005	1.507011595	0.893980258	1.41
5	1.2005	1.51678903	0.884202823	1.39
6	1.2005	1.639741695	0.761250158	1.46
7	1.2005	1.500557911	0.900433942	1.19
8	1.2005	1.532534517	0.868457335	1.37
9	1.2005	1.525958845	0.875033008	0.95
10	1.2005	1.507011595	0.893980258	1.26
11	1.2005	1.550892833	0.850099019	1.45
12	1.2005	1.59059276	0.810399092	1.17
13	1.2005	1.537736483	0.86325537	1.26
14	1.2005	1.524375074	0.876616779	1.13
15	1.2005	1.509712226	0.891279627	1.12
16	1.2005	1.55702269	0.843969162	1.08
17	1.2005	1.527566079	0.873425774	1.39
18	1.2005	1.707693252	0.693298601	1.43

19	1.2005	1.534241668	0.866750185	1.25
20	1.2005	1.543190862	0.857800991	1.17
21	1.2005	1.529197361	0.871794491	1.31
22	1.2005	1.507011595	0.893980258	1.03
23	1.2005	1.530853298	0.870138554	0.94
24	1.2005	1.635871613	0.76512024	1.54
25	1.2005	1.548918751	0.852073102	1.20
26	1.2005	1.527566079	0.873425774	1.04
27	1.2005	1.498088223	0.90290363	1.17
28	1.2005	1.521275681	0.879716172	1.26
29	1.2005	1.532534517	0.868457335	1.02
30	1.2005	1.674935581	0.726056271	1.25

u - Chart: